Lecture 1

Tuesday, September 1, 2020

Something to ponder:

* Why are the doctrines of the Gospel so easy to understand while the concepts of math, although less important, so difficult to understand?
* learning things pertaining to God's creation is a commandment (D&C 88, verse 77–79).

Overview of the course.

Linear Algebra is in several ways parallel to Calculus I. The main object we worked with Calc. I is functions, specifically functions of one variable. We learned properties of a function through its graph (which is a curve). The graphs motivate important concepts of analysis such as limit, continuity, derivative and integral.

In Linear Algebra, we will learn functions of more than one variable. But we only consider a special type of functions called linear functions. A function, say $f(x,y,z)$ of three variables, is called linear if f has the form

$$f(x,y,z) = ax + by + cz$$

where a, b, c are constants. Terms such that x^2, xy, $\sin x$, $y \sin x$, ... are not allowed to appear in the expression of f

The graph of a two-variable function $f(x,y)$ is a surface in the space. For example, function $f(x,y) = x^2 + y^2$ (not a linear function) has a graph of paraboloid shape.

One needs to render the graph of function $f(x,y,z)$ in a four-dimensional space, which is not possible to draw. Thus, the one can't study properties of multivariable functions through its graph as we did to single-variable functions in Calc. I However, for linear multivariable functions, there is a substitute for graphs.

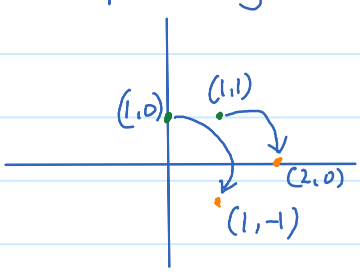
Each linear function is associated with a matrix. For example, function $f(x,y) = 2x + 3y$ is associated with matrix $A = [2 \ 3]$. We will explain why later. A matrix is a compact form of information capable of fully represent a linear map, just like the fact that the graph of a one-variable function can fully represent the function itself. A matrix of a linear function can thus serve as the "graph" of that function.

One can learn many properties of a linear map through its matrix. For example,

- Algebraic operations: how to add, subtract, multiply, divide two functions.
- Determinant: how much a linear function stretches a shape
- Eigenvectors and eigenvalues: special directions unchanged by a linear function
- Singular value decomposition. a helpful representation of a linear function

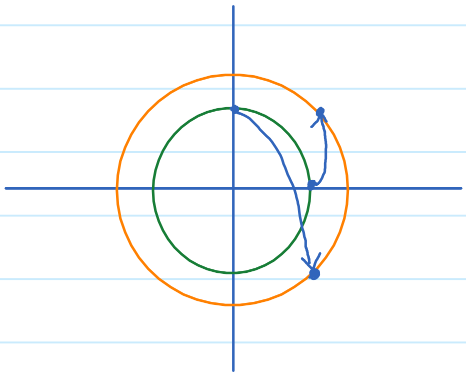
* More comments on determinant:

Let us consider a function $f(x,y) = (x+y, x-y)$. This is a linear function. (We will explain why later.) It takes point (x,y) on the plane to point $(x+y, x-y)$ on the plane. In this way, f can be viewed as a "geometric transformation."



One can ask what the unit circle $\mathcal{C} = \{(x,y) : x^2+y^2=1\}$ is transformed into. One can see that the point $(x+y, x-y)$ lies on the circle of radius 2 because

$$(x+y)^2 + (x-y)^2 = 2(x^2+y^2) = 2.$$



The area of the unit circle is π. The area of the circle after being transformed is 2π. The area is thus stretched by two times. The determinant of f is equal to -2. (The minus sign is due to the change of orientation of the unit circle after being transformed.)

We will see that this linear map is associated with matrix

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$

We will also see that it is much easier to compute the determinant of A than to compute the stretch of area